New bounds and constructions for neighbor-locating colorings of graphs

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Neighbor-locating coloring

Introduction

- Behtoei et al. (2014) → Adjacency-locating coloring.
- Alcon et al. (2019) → Neighbor-locating coloring.

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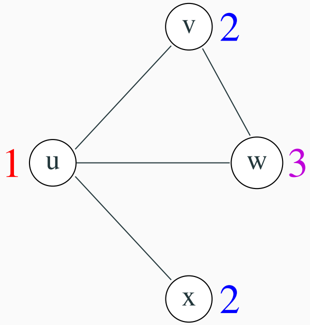

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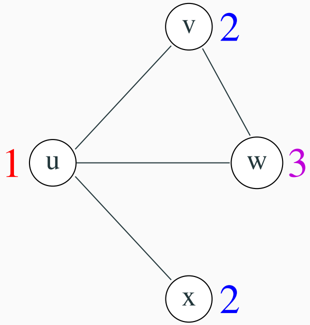

Color neighbors of $u \rightarrow \{2,3\}$

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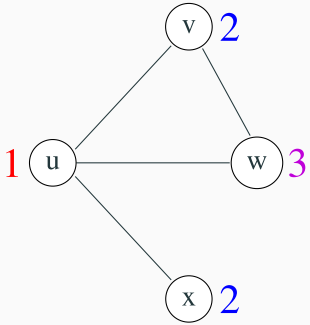

Color neighbors of $u \rightarrow \{2,3\}$

Color neighbors of $x \rightarrow \{1\}$

Neighbor locating k-coloring

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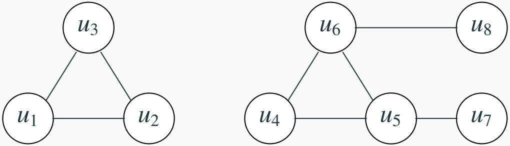

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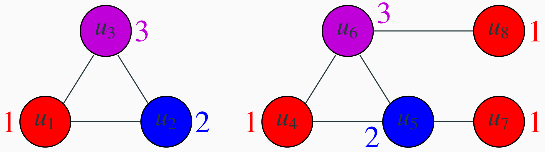

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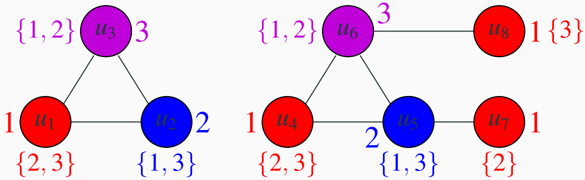

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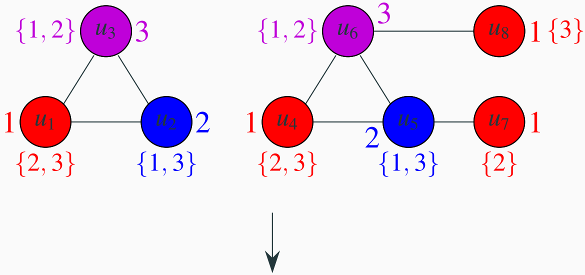

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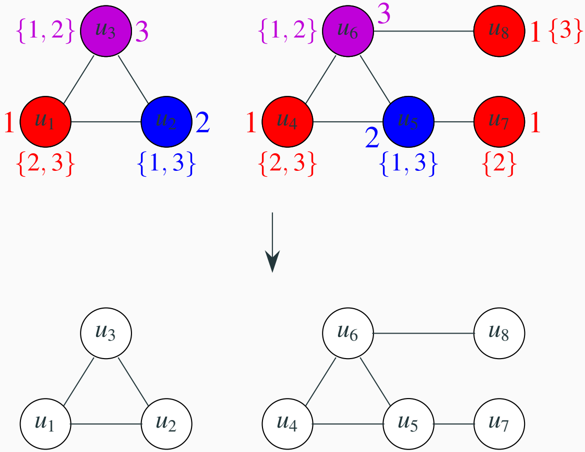

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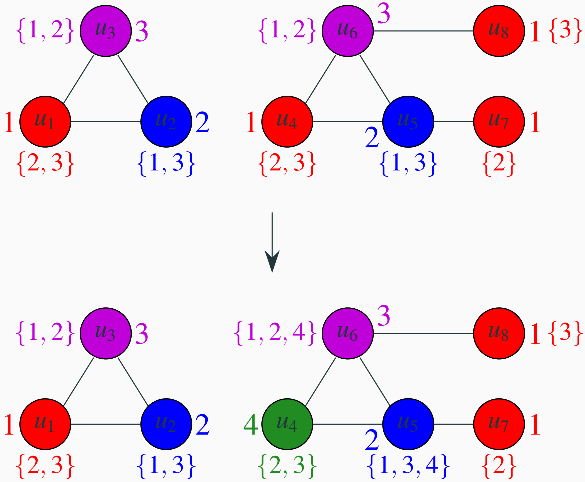

Neighbor-locating chromatic number

$$\chi_{NL}(G) = min\{k : G \text{ admits a neighbor-locating } k\text{-coloring}\}.$$

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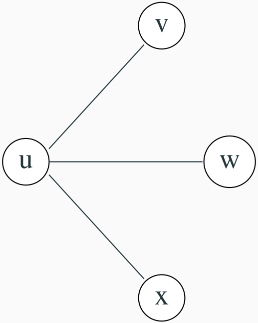

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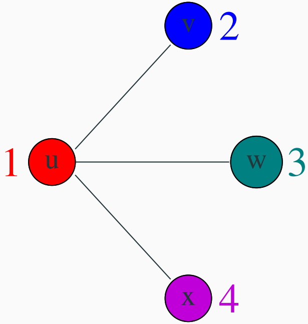

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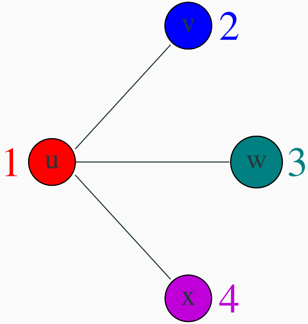
Example: $\chi_{NL}(G) = 4.$



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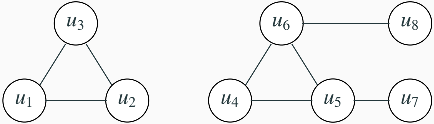

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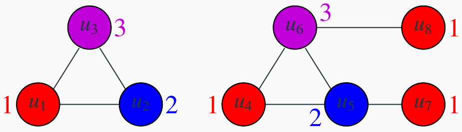

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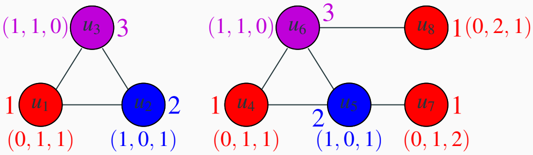

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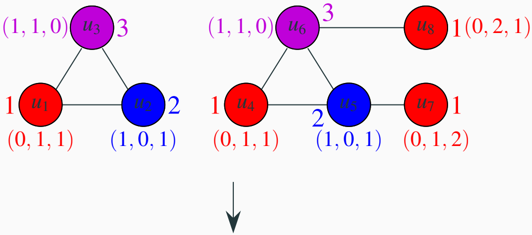

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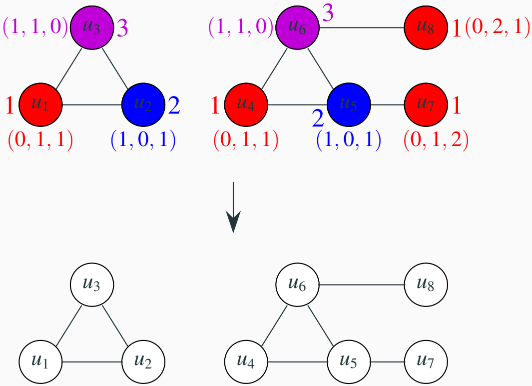

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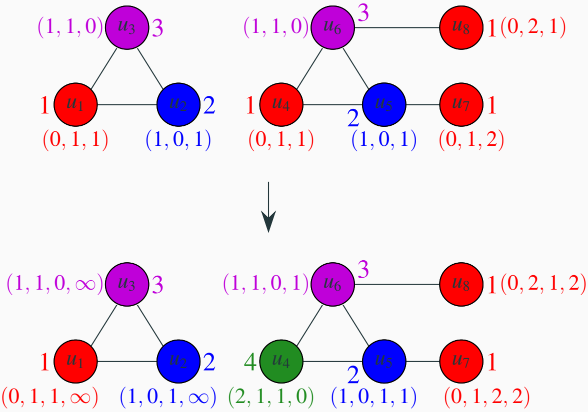

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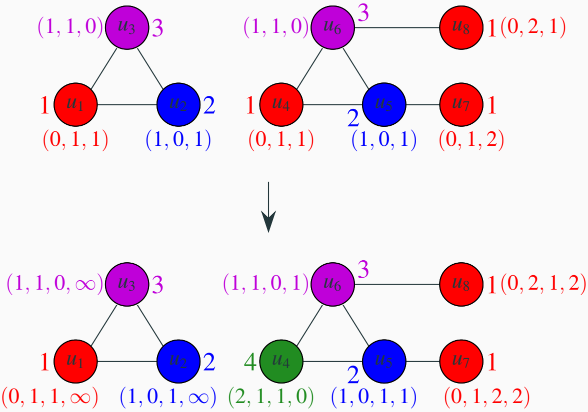

Locating chromatic number:

$\chi_L(G) = min\{k : G$ admits a locating k-coloring$\}$.

Comparing ordinary coloring, locating coloring and neighbor-locating coloring

Observations

False twins receive different colors under a neighbor-locating coloring and locating coloring.

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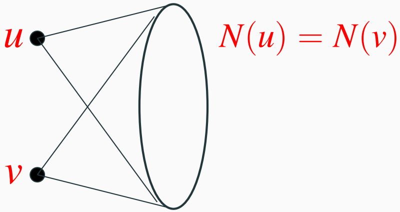

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Every neighbor-locating coloring is a locating coloring.

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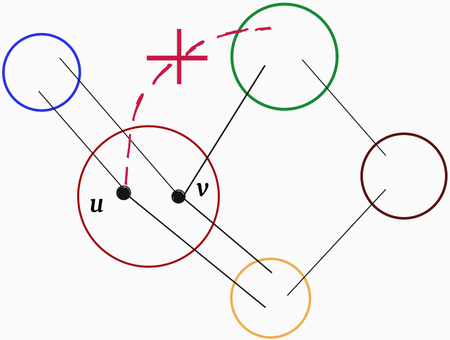

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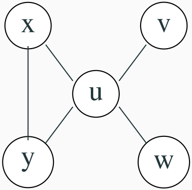

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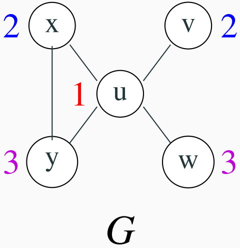

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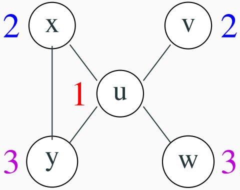

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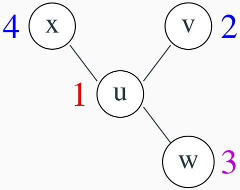

H

Theorem 1

For every $k \geq 0$, there exists a graph G_k having an induced subgraph H_k such that $\chi_L(H_k) - \chi_L(G_k) = k$ and $\chi_{NL}(H_k) - \chi_{NL}(G_k) = k$.

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Theorem 2
For all $2 \leq p \leq q \leq r$, except when $p = q = 2$ and $r > 2$, there exists a connected graph $G_{p,q,r}$ satisfying $\chi(G_{p,q,r}) = p$, $\chi_L(G_{p,q,r}) = q$, and $\chi_{NL}(G_{p,q,r}) = r$.

Bounds and constructions for sparse graphs

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Examples:
(1) Trees $\rightarrow$ $(n-1)$ edges.
(2) Planar graphs $\rightarrow$ at most $(3n-6)$ edges.

Bound for sparse graphs

Theorem 3

Let G be a connected graph on n vertices and m edges such that $m \leq an + b$, where $2a$ is a positive integer and $2b$ is an integer. If $\chi_{NL}(G) = k$, then

$$n \leq 2b + k \sum_{i=1}^{2a} (2a + 1 - i) \binom{k-1}{i}.$$

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Connected graph	$k(2^{k-1}-1)$ [Alcon et al.]

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Unicyclic	$\frac{k^3+k^2-2k}{2}$

Asymptotic tightness of the obtained bound

Theorem 4

Let $2a$ be a positive integer and let $2b$ be an integer. Then, there exists a graph G on n vertices and m edges satisfying $m \leq an + b$ such that $n = \Theta(k^{2a+1})$ and $\chi_{NL}(G) = \Theta(k)$. Moreover, when $b = 0$, G can be taken to be of maximum degree $2a$.

Open problems

1. Construct graphs which exactly attain the bound.
2. Characterize the graphs for which $\chi(G) = \chi_{NL}(G)$.
3. Characterize the graphs for which $\chi_L(G) = \chi_{NL}(G)$.
4. Find the neighbor-locating chromatic number for trees.

THANK YOU